The Mathematical Crusade

The Mathematical Relay

SECTION A

Q1. a) The symbol n! is used to represent the product n(n–1)(n – 2) ... (3)(2)(1). For example,4!= 4(3)(2)(1). Determine n such that $n!=(2^{15})(3^{6})(5^{3})(7^{2})(11)(13)$.

b) The numbers 1, 2, 3, . . . , 9 are placed in a square array. The sum of the 3 rows, the sum of the 3 columns, and the sum of the 2 diagonals are added together
to form a "grand sum", S.
For example, if the numbers are placed as shown, the grand sum is

1	2	3
4	5	6
7	8	9

S = row sums + column sums + diagonal sums
= 45 + 45 + 30
= 120 .
What is the maximum possible value of the grand sum S?

Q2. Three people, A, B and C are being judged for a crime. Each of these people could or could not come from the island. Everyone that comes from the island is either a liar or a truth-teller. People that do not come form the island are called normal and might or might not tell the truth.

It is known that the crime was committed by one of the three suspects. It is also known that the person who committed the crime is a truth-teller, and is the only truth-teller among the three suspects. The suspects said:

A: "I am innocent."
B: "That is correct."
C: "B comes from the island."

Who is the guilty one?

Q3. You are given a solid 1x1x1 cube. An ant is placed at one corner. What is the minimum distance it can cover so as to reach the diagonally opposite corner?

Q4. Find the area of the largest semi-circle that can be made inside a square of side 1.

Q5. Two trains are approaching each other with a speed of 50kmph. A bird starts flying from the windscreen of one train to that of the other, when they are at a distance of 200m. It then goes back to the first train, and so on till the two trains collide. The bird moves at a constant speed of 20kmph. What is the total distance covered by the bird?

Q6. Prove that $\frac{a+b+c}{3} \geq \sqrt[3]{abc}$

where a, b, c are positive real numbers.

SECTION B

1. Prove that there is no prime number p>3 such that 2^p+1 is divisible by p.

2. Find all integers such that n^2+1 is divisible by n+1.

3. Prove that {(2000.1998.1996.....2) – (1.3.5.7.....1999)} is divisible by 2001.

4. Is there an infinite increasing geometric progression such that the first 100 elements of the sequence but none of the rest of the elements are integers? If yes, give an example.

5. Find the number of n-character strings that can be formed using the alphabets A,B,C,D,E such that each string has an even number of A's

6. A function f is continuous in the interval (0.51,0.67). It takes rational values over this interval. Given that f(0.55) = 5, find f(π/5).

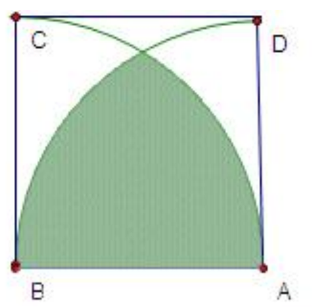
SECTION C

1. A set of numbers has "the triple-sum property" (or TSP) if there exist three numbers in the set whose sum is also in the set. [Repetitions are allowed.] For example, the set $U = \{2, 3, 7\}$ has TSP since $2 + 2 + 3 = 7$, while $V = \{2, 3, 10\}$ fails to have TSP.

Suppose the set {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12} is separated into two parts, forming two subsets *A* and *B*.
Prove: Either *A* or *B* must have the triple-sum property.

2. There are 9 people in a party. A person speaks either English or Latin. (NO person speaks both, and every person exactly one language). Every person is wearing either a black or a white hat. Prove that there are atleast 3 people wearing the same colour hat AND speaking the same language.

3. In the given figure, ABCD is a unit square. A and B are the centers of the circles. Find the area of the shaded region.



4. Three tablespoons of milk from a glass of milk are poured into a glass of tea. The mixture is thoroughly mixed. Now three tablespoons of this mixture are poured back into the glass of milk. What is greater now, the percentage of milk in the cup of tea or the percentage of tea in the glass of milk? Explain.