

# The Mathematical Crusade

## Senior Quiz Prelims

Q1. Find the integral solutions of  $x$  and  $y$  in the following equation:

$$1/x + 1/y = 1/12$$

(5 marks)

Q2. The absolute value of a real number  $x$ , denoted as  $|x|$  is the numerical magnitude of the number, irrespective of its sign, while the maximum of two real numbers  $x$  and  $y$ , denoted as  $\max(x,y)$  is the number that is numerically bigger.

So, for example,  $|-2| = |2| = 2$  and  $\max(-3,2) = 2$ .

a) Express  $|x|$  as a function involving at most the maximum, addition, subtraction, division and multiplication.

b) Express  $\max(x,y)$  as a function involving at most the absolute value, addition, subtraction, division and multiplication.

(5 marks)

Q3. There exists a unique 10-digit natural number, such that the first digit equals the number of 0's in the number, the second digit the number of 1's, the third digit the number of 2's and so on. Find the number.

(5 marks)

Q4. Can one form a magic sq with 1<sup>st</sup> 81 prime no's? Give reasons for your answer.

(5 marks)

Q5. Show that a triangle is equilateral *if and only if*:

$$\tan A + \tan B + \tan C = 3\sqrt{3}$$

(5 marks)

Q6. Identify the speaker of the lines "God does not care about our mathematical difficulties. He integrates empirically."

(5 marks)

Q7. A king just received 1023 bottles of wine, for a party he has to host the next day. He has just received news that due to a management error, one of the bottles is poisoned. He does not know which one, but is sure that's there's only one poisoned bottle. The poison is exceptionally fatal, and hence cannot get diluted. Furthermore, the effects of the poison take one month to surface. The king decides he will get some of his prisoners in his vast dungeons to drink the wine. At a minimum how many prisoners does he need to kill to find out the poisoned bottle of wine, so that he has no casualty amongst his guests? (He can't reorder the wines)

(8 marks)

Q8. A high school has a very strange principal. On the first day, he has his students perform an odd opening day ceremony. There are one thousand lockers and one thousand students in the school. The principal asks the first student to go to every locker and open it. Then he has the second student go to every second locker and close it. The third goes to every third locker and, if it is closed, he opens it, and if it is open, he closes it. The fourth student does this to every fourth locker, and so on. After the process is completed with the thousandth student, how many lockers are open?

(8 marks)

Q9. Write the next two terms in the following sequences:

a) 2, 6, 12, 20, 35, 66...

b) 1, 2, 6, 20, 70...

(2x4 =8 marks)

Q10. a) Prove 111...1 ninety-one times is not a prime number.

b) Is the number 2438100000001 prime or composite? How can you say this?

(2x4=8 marks)

Q11. Show that

$$1 + x + x^2/2! + x^3/3! + \dots + x^{2n}/(2n)!$$

is positive for all real values of x

(10 marks)

Q 12. Find the volume if the area bounded by the curve  $y = x^3 + 1$ , the x-axis and the limits of  $x = 0$  and  $x = 3$  is rotated around the x-axis.

(10 marks)

Q13. Find the following limit, or show that the limit doesn't exist.

$$\lim_{(x,y) \rightarrow (0,2)} \frac{6x^3 \sqrt{y-2}}{2x^4 + y^2 - 4y + 4}$$

(10 marks)

Q14. The expression for Kinetic energy under special relativity is given by

$$K = \gamma mc^2 - mc^2$$

where,

$$\gamma = \frac{1}{\sqrt{1 - u^2/c^2}}$$

and  $m$ ,  $c$  and  $u$  are rest mass, speed of light and speed of mass respectively. Show that this is nearly equivalent to the Newtonian value for kinetic energy at low speeds.

(10 marks)