THE MATHEMATICAL CRUSADE '10

Four Questions

- There are 12 questions in total, each worth 10 points.
- At any given point of time, you will not have more than 4 questions to attempt.
- If you manage to solve a question, submit the answer immediately.
- If you believe you cannot solve a question, you are welcome to pass it.
- Once you attempt or pass a question you will be given the next question, until the event ends or we run out of questions.
- All questions are problem solving questions. You are required to write complete solutions with proof.
- Ensure that your school name and school registration number is clearly written and marked on every sheet you submit.
- If you have any further queries, do not hesitate to ask the student volunteer or the teacher in-charge in your classroom.

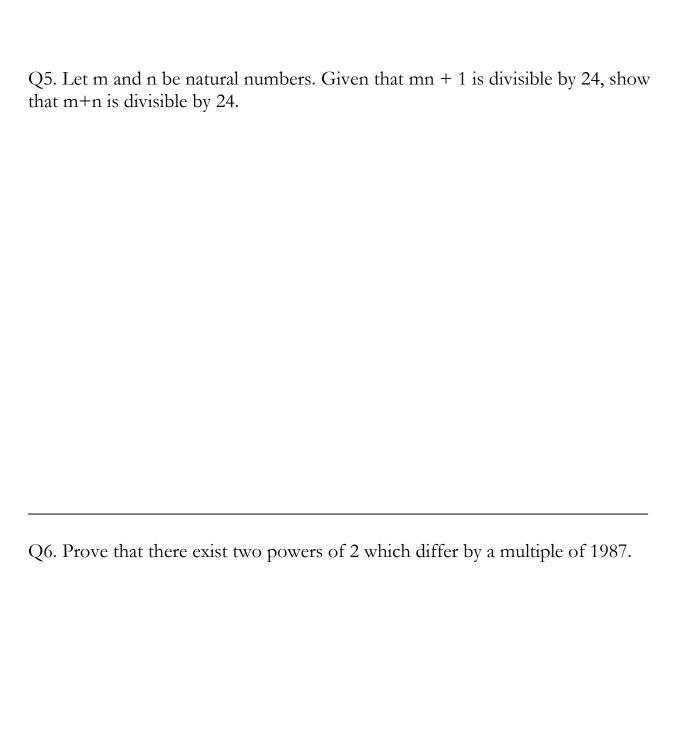
Q1. Nine numbers are placed around a circle: four 1s and five 0s. The following operation is performed on the numbers: between each adjacent pair of numbers is placed a 0 if the numbers are different and a 1 if the numbers are the same. The 'old' numbers are then erased. After several of these operations, can all the remaining numbers be equal?

Q2. S(k)=1!+2!+3!+...+k!For what values of k is S(k) a perfect square? Q3. Prove that a plane can be perfectly tiled using an infinite number of congruent regular triangles, squares and hexagons. Further prove that it cannot be tiled by any other regular polygon.

Q4. A play has the following rules:

- There are fruits kept at integral points on the coordinate plane bounded by the positive x and y axis. There is one poisonous fruit at (0,0).
- If a player eats a fruit kept at (x,y), all fruits at points (a,b) where a>=x, and b>=y are damaged, i.e. the second player cannot choose them anymore.
- The game finishes when one of the players dies, i.e., eats the poisonous fruit.

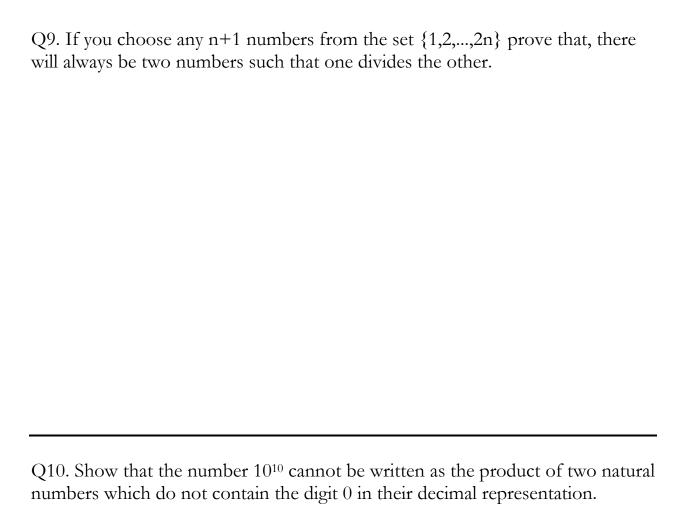
Prove that the both players cannot survive forever.

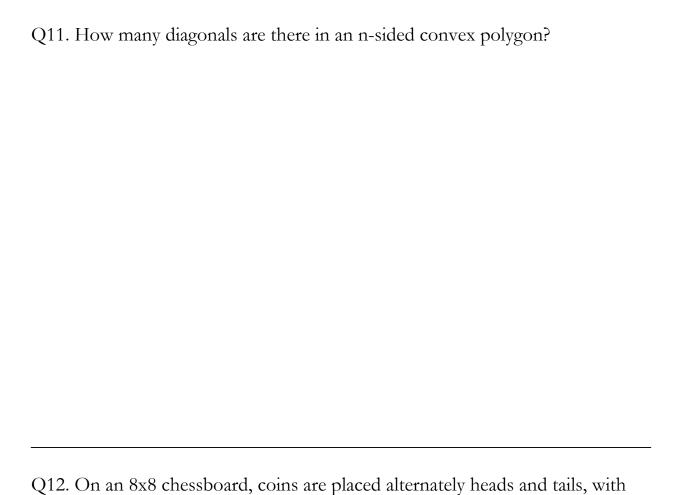


Q7. Goldbach's strong conjecture states that every even integer greater than 4 can be expressed as the sum of 2 primes, not necessarily distinct. Goldbach's weak conjecture states that every odd number greater than 7 can be expressed as the sum of 3 primes. Show that Goldbach's weak conjecture follows from his strong conjecture!?

Q8. Show that the following inequality holds for all positive real numbers a, b, and c:

 $(a+b+c)\left(\frac{1}{a+b}+\frac{1}{b+c}+\frac{1}{c+a}\right)\geq \frac{9}{2}$





heads on black squares and tails on white squares. In a given step, any two coins

are chosen and flipped. Can there, after any number of steps, remain only 1

head on the chessboard?