The Mathematical Crusade '10

Junior Quiz Prelims

- This is the preliminary round of the Junior Quiz. The 6 top-scoring teams will qualify for the final on-stage round.
- In case Delhi Public School, R.K. Puram qualifies, the top 7 teams will qualify for the final on-stage round.
- The winners, 1st runners-up, and 2nd runners up of the final round will be awarded 100, 80 and 60 points respectively that shall count towards the overall tally.
- Qualifying teams that do not place will be awarded 30 points towards the overall tally.
- There are 17 questions in this paper, worth 90 points in a total of two printed sides.
- Questions 1-2 are Trivia questions each of 5 points, worth a total of 10 points. Please note that Q1 and Q2 are the Clueless clues in the Junior Quiz.
- Questions 3-5 are Trivia questions worth a total of 15 points.
- Questions 5-17 are Problem Solving questions worth a total of 65 points.
- Kindly show the working for the problem solving questions.
- You will be given blank sheets of paper to write your answers and solutions in. Affix the space for the school and participants' names with your answer sheet.
- Ensure that your school name and school registration number is clearly written and marked on every sheet you submit.
- If you have any further queries, do not hesitate to ask the student volunteer or the teacher in-charge in your classroom.

Q1. The statement of which conjecture is given as "The real part of any non-trivial zero of the Riemann zeta function is ½"!?

(5 Points)

Q2. The mathematician Christian Goldbach, born in Königsberg, a part of Brandenburg-Prussia wrote a letter to Leonhard Euler in the year 1742. What is the historical significance of this letter!? (5 Points)

Q3. Which prize in mathematics is awarded only to mathematicians below the age of 40? (5 Points)

Q4. It is known as "Principio dei Cassette" in Italian, and called the "Dirichlet Principal" in Russia, as it was first formalized by Johann Dirichlet in 1834 under the name *Schubfachprinzip* ("Drawer Principle" or "Shelf Principle"). What is the principle more commonly known as? (5 Points)

Q5.

"*Now I defy a tenet gallantly*
Of circle canon law: these integers
Importing circles' quotients are, we see,
Unwieldy long series of cockle burs
Put all together, get no clarity;
Mnemonics shan't describeth so reformed
Creating, with a grammercy plainly,
A sonnet liberated yet conformed.
Strangely, the queer'st rules I manipulate
Being followéd, do facilitate
Whimsical musings from geometric bard.
This poesy, unabashed as it's distressed,
Evolvéd coherent - a simple test,
Discov'ring poetry no numerals jarred."

The construction of such 'rhymes', falls under a certain branch of linguistics, named as a type of ______philology. What does this refer to?

(5 Points)

Q6. The sequence of numbers 1, 2, 3, ..., 2010 are placed on a checkerboard. Two randomly chosen numbers are removed from the board, and the difference between the two numbers is replaced on the checkerboard. This process is continued until only one number remains on the checkerboard. Can this number be 0?
(5 Points)

Q7. Given natural numbers a, b and c such that a + b + c is divisible by 6, prove that $a^3 + b^3 + c^3$ is divisible by 6. (4 Points)

Q8. Prove that the fraction $\frac{12n+1}{30n+2}$ cannot be reduced further for any natural number n. (4 Points)

Q9. How many ways are there to split 14 people into seven pairs?
(6 Points)

Q10. Let $P(x) = x^4 + x^3 + x^2 + x + 1$. What is the remainder when $P(x^{12})$ is divided by $P(x)$? (7 Points)

Q11. Prove that, for any natural number n>1, there exists an infinite number of strings of n consecutive composite numbers. (5 Points)

Q12. Given the pair of prime numbers p and p^2+2, prove that p^3+2 is also a prime number. (7 Points)

Q13. Show that a square number is of the form 8k+1, for integral k, if and only if k is a triangular number. (4 Points)

Q14. Show that any positive integer can be expressed as a unique sum of powers of 2. For example, 5=4+1, 179 =128+32+16+2+1, and 8=8.
(5 Points)

Q15. In a district, there are n villages. From each village there are at least (n-1)/2 roads connecting it to other villages. Prove that a path always exists between any two villages. (6 Points)

Q16. Let $f(x) = ax^2 + bx + c$. Suppose $f(x) = x$ has no real roots. Show that the equation $f(f(x)) = x$ has no solutions. (5 Points)

Q17. p is a prime such that p^2 is of the form $a^2 + 2b^2$ where a and b are positive integers. Prove that p is also of this form (7 points)