

THE MATHEMATICAL CRUSADE '10

Puzzle Hut

- There are 16 questions in the paper.
- You have to submit the answers on sheets of paper given to you. You may ask for more at the Help Desk.
- All participants from every school are expected to contribute for this event.
- You can get a maximum score of 200 points.
- The paper must be submitted to the Help Desk in the AVH by the end of the Senior Surprise Event finals.
- There will be no trophy for this event; however 50, 40 and 30 points respectively will be awarded to the winners, 1st runners-up, 2nd runners-up respectively, which will count towards the overall tally.
- You are not permitted to use any electronic items when the Puzzle Hut is in progress.
- Please note, there are no Clueless clues in this paper.
- You are not allowed to take any help from your teacher in-charge, if any.
- For any queries, please contact the help desk.

Q1. Philomath sits down at his computer at twelve o'clock and starts his English assignment. He types the first four sentences and then discovers to his amazement that a strange virus seems to have affected his computer. For even though he types in English and according to the correct letters as denoted by his keyboard, a baffling array of letters appear. Below is a transcript of what he typed:

Sicop n ivuykcejhiyu h adwxes'uged?

He has forgotten what he has written so he tries to decipher the code to avoid having to start again. After one hour, he still hasn't figured it out. He then looks at the clock on the wall and decodes his assignment.

What is the virus doing and what does the above say? (20 points)

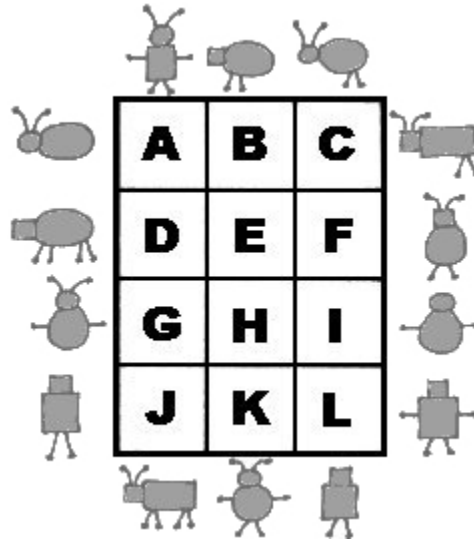
Q2. You are blindfolded and let into a room. The room has an infinitely many coins scattered around on the floor. Your friend tells you that that 20 of these coins are tails and the rest are heads. He also says that if you can divide the coins into 2 piles where the number of tails is the same in both piles, then you win all of the coins. You are allowed to move the coins and to flip them over, but you can never tell what state a coin is currently in (the blindfold prevents you from seeing, and you cannot tell by feeling it). How do you go about partitioning the coins so that you can win all of them? (10 points)

Q3. As the pirates rowed Tainted Angel, their ship, away from the Greek island where they had rested, left behind in the sand was a message (shown below) that a shipmate had written about their captain, Short John Sliver. What did it say?

ο Δ ρ Γ ι Σ δ

(20 points)

Q4. Shown right are 12 mutation chambers labeled A-L. Each of the four aliens on the left passes through the three chambers directly to the right to finish as the alien on the right (e.g. the alien to the left of A passes through A, B, C to finish as the one to the right of C). Also, each of the three aliens at the top passes through the four chambers directly below to finish as the alien at the bottom. Every chamber performs a mutation and does so on one part of the body (e.g. makes head square). What does each of the 12 chambers do?



(10 points)

Q5. Decapitate me and all becomes equal. Then truncate me and I become second. Cut me front and back and I become two less than I started.

What am I?

(15 points)

Q6. Player A has one more coin than player B. Both players throw all of their coins simultaneously and observe the number that come up heads. Assuming all the coins are fair, what is the probability that A obtains more heads than B?

(10 points)

Q7. The number 60 is written on a blackboard. You and you friend take turns in subtracting from the number on the blackboard any of its divisors, and replacing the original number with the result of this subtraction. The player who writes the number 0 loses. Devise a winning strategy for this game.

(15 points)

Q8. Five men crash-land their airplane on a deserted island in the South Pacific. On their first day they gather as many coconuts as they can find into one big pile. They decide that, since it is getting dark, they will wait until the next day to divide the coconuts. That night each man took a turn watching for rescue searchers while the others slept. The first watcher got bored so he decided to divide the coconuts into five equal piles. When he did this, he found he had one remaining coconut. He gave this coconut to a monkey, took one of the piles, and hid it for himself. Then he jumbled up the four other piles into one big pile again. To cut a long story short, each of the five men ended up doing exactly the same thing. They each divided the coconuts into five equal piles and had one extra coconut left over, which they gave to the monkey. They each took one of the five piles and hid those coconuts. They each came back and jumbled up the remaining four piles into one big pile. What is the smallest number of coconuts there could have been in the original pile? (20 points)

Q9. Prove that there exists a power of 3 which ends with the digits 001 in decimal notation. (10 points)

Q10. Show that in any group of 5 people, there are two who have an identical number of friends within the group. (10 points)

Q11. What is the smallest positive integer that leaves a remainder of 1 when divided by 2, remainder of 2 when divided by 3, a remainder of 3 when divided by 4, ... and a remainder of 9 when divided by 10? (10 points)

Q12. Prove that powers of 2 cannot be expressed as a sum of any k consecutive integers. (10 points)

Q13. You have two very hungry termites and two sticks of wood. One stick of wood is 12 inches long and the other is 16 inches long. One termite can eat sticks at the rate of 1 inch every 3 minutes. The other termite can eat 1 inch in 4 minutes. How would you use the termites and sticks to measure 61 minutes? (10 points)

Q14. In the Art Gallery is a new conceptual art piece. The artist created a huge pile of different paintings; periodically he picks up one of the paintings and hangs it on the wall, and adjusts it.

At 11:00, the painting titled "Number 30" was hung.

At 4:00, the painting titled "Number 240" was hung.

At 7:30, the painting titled "Number 315" was hung.

What painting will the artist hang on the wall at 9:20? (10 points)

Q15. Using only two 2's and any combination of mathematical signs, make a total of 5. (10 points)

Q16. Given five zeroes, using any mathematical operations, make a total of 120. (10 points)