THE MATHEMATICAL CRUSADE '10

Senior Quiz Prelims

- This is the preliminary round of the Senior Quiz. The 6 top-scoring teams will qualify for the final on-stage round.
- In case Delhi Public School, R.K. Puram qualifies, the top 7 teams will qualify for the final on-stage round.
- The winners, 1st runners-up, and 2nd runners up of the final round will be awarded 100, 80 and 60 points respectively that shall count towards the overall tally.
- Qualifying teams that do not place will be awarded 30 points towards the overall tally.
- There are 14 questions in this paper, worth 80 points in a total of two printed sides.
- Questions 1-4 are Trivia questions worth a total of 20 points.
- Questions 5-13 are Problem Solving questions worth a total of 50 points.
- Question 14 is a Problem Solving question worth 10 points. Please note that Q14 is the Clueless clue in the Senior Quiz.
- Kindly show the working for the problem solving questions.
- You will be given blank sheets of paper to write your answers and solutions in. Affix the space for the school and participants' names with your answer sheet.
- Ensure that your school name and school registration number is clearly written and marked on every sheet you submit.
- If you have any further queries, do not hesitate to ask the student volunteer or the teacher in-charge in your classroom.

Q1. Which prize in mathematics is awarded only to mathematicians below the age of 40? (3 Points)

Q2. It is known as "Principio dei Cassette" in Italian, and called the "Dirichlet Principal" in Russia, as it was first formalized by Johann Dirichlet in 1834 under the name *Schubfachprinzip* ("Drawer Principle" or "Shelf Principle"). What is the principle more commonly known as? (5 Points)

Q3.

"*Now I defy a tenet gallantly*
Of circle canon law: these integers
Importing circles' quotients are, we see,
Unwieldy long series of cockle burs
Put all together, get no clarity;
Mnemonics shan't describeth so reformed
Creating, with a grammercy plainly,
A sonnet liberated yet conformed.
Strangely, the queer'st rules I manipulate
Being followéd, do facilitate
Whimsical musings from geometric bard.
This poesy, unabashed as it's distressed,
Evolvéd coherent - a simple test,
Discov'ring poetry no numerals jarred."

The construction of such 'rhymes', falls under a certain branch of linguistics, named as a type of ______philology. What does this refer to?

(5 Points)

Q4. A mathematical theorem, whose proof has been accepted recently, was stated on the satirical news show, the Colbert Report by Stephen Colbert as "Rabbits are Spheres, because they don't have any holes." Which celebrated conjecture does this refer to?

(7 Points)

Q5. Given a 5x5 chessboard, and any one square removed, can you cover the remaining with 3x1 tiles? Which squares can be removed? Prove your answer. (5 Points)

Q6. Prove that the polynomial formed by truncating the Taylor expansion of e^x up to the (2n+1)th term is always positive. (7 Points)

Q7. z_1, z_2, z_3, z_4 are 4 distinct complex numbers, such that when plotted on the Argand plane they form a quadrilateral's vertices taken clockwise. Find a necessary and sufficient condition for z_1, z_2, z_3 and z_4 such that the formed quadrilateral is a parallelogram. (4 Points)

Q8. In a district, there are n villages. From each village there are at least (n-1)/2 roads connecting it to other villages. Prove that a path always exists between any two villages. (5 Points)

Q9.Prove that there exist infinitely many natural numbers n such that n, n+1, n+2 are all sums of squares of two integers. (7 Points)

Q10. p(x)is a polynomial with integral coefficients. Prove that if both p(0) and p(1) are odd, then p(x) has no integral roots. Will the result hold if p(0) and p(-1) are odd? Will it hold if p(-1) and p(1) are odd? (5 Points)

Q11. There are many ordered integral quadruples satisfying $a + b + c + d = 15$. Find the probability that $a > 1$. (6 Points)

Q12. Define $f(\alpha)$ as:

$$f(\alpha) = \int_0^\infty \frac{1}{(1+x^2)(1+x^\alpha)}\,dx$$

Determine $f'(\alpha)$. And hence find $f(2010)$. (7 Points)

Q13. If f(x) is a polynomial such that f(x) = x has no real solutions, show that the equation f(f(x)) = x has no real solutions. (4 Points)

Q14. Are there infinitely many primes of the form n^2+1 !?
(10 Points)